Reply to Comment on "Quantification of Macroscopic Quantum Superpositions within Phase Space"

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Our recent Letter [1] proposes a general measure, $\mathcal{I}$, to quantify macroscopic quantum superpositions. Gong points out [2] a "direct connection" between $\mathcal{I}$ and a previously studied quantity, χ^2, introduced as a measure of "phase-space distributional heterogeneity" [3] to study classical and quantum chaos [3, 4]. He comments that $\mathcal{I}$ and χ^2 are basically equivalent for pure states, and "closely related" also for mixed states.

While it is beneficial to learn some unrecognized connection between the two measures, as we shall see, $\mathcal{I}$ and χ^2 are very different for some mixed states, and χ^2 does not work as a sensible measure to quantify quantum superpositions. In fact, χ^2 has never been considered as a quantifier of quantum superpositions in the literatures [3, 4].

According to the decohrence model considered in Ref. [1], $\mathcal{I} = -\dot{\mathcal{P}}/2$ corresponds to "purity decay rate" while $\chi^2 = 2(1 - \dot{\mathcal{P}}/\mathcal{P})$ relates to "purity decay rate divided by purity", where $\mathcal{P}$ and $\dot{\mathcal{P}}$ denote the purity and its time derivative, respectively. In what follows, we present two specific examples to highlight the crucial difference between the two measures.

We first consider a superposition of thermal states introduced in Ref. [5]:

$$\rho_M \propto \rho_{th}(V,d) + \rho_{th}(V,-d) + \sigma(V,d) + \sigma(V,-d) \quad (1)$$

where $\rho_{th}(V,d) = \int d^2\alpha P_{th}(V,d)|\alpha\rangle\langle\alpha|$ and $\sigma(V,d) = \int d^2\alpha P_{th}(V,d)|\alpha\rangle\langle-\alpha|$ with coherent state $|\alpha\rangle$ of amplitude α and $P_{th}(V,d) = \frac{2}{\pi(V-1)}\exp[-\frac{2|\alpha-d|^2}{V-1}]$. Here, V and d respectively correspond to variance and displacement of the displaced thermal state $\rho_{th}(V,d)$. This type of state plays an important role in revealing some critical quantum behaviours [5, 6]. It is straightforward to obtain $\mathcal{I}$ and χ^2 for ρ_M based on Refs. [1] and [3], and the results for $d = 1$ are plotted in Fig. 1. Obviously, $\mathcal{I}$ and χ^2 behave in opposite ways: when V increases, $\mathcal{I}$ decreases to approach 1/2 [7] but χ^2 keeps increasing without limitation.

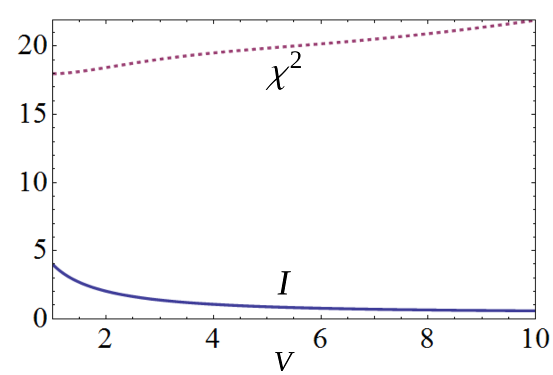

FIG. 1: Plots of $\mathcal{I}$ (solid curve) and χ^2 (dotted curve) against V for state ρ_M with $d = 1$.

The second example further clarifies what happens. Let us consider a mixed state:

$$\rho_m = p|1\rangle\langle1| + (1-p)\rho_{th}(V,0), \quad (2)$$

where $|1\rangle$ is a single photon state and $V \to \infty$ is assumed [8]. We then observe that χ^2 of ρ_m for an *arbitrarily* small value of p approaches 6: this value of χ^2 equals that of the single photon and far larger than that of a thermal state $\chi^2 \approx 0$ for $V \to \infty$. When p is very small, ρ_m approaches a thermal state of $V \to \infty$ (or the identity operator), *i.e.*, states ρ_m and $\rho_{th}(V \to \infty, 0)$ become nearly identical both theoretically and practically. Therefore, such an asymptotic behaviour of χ^2 does not make sense as a measure of quantum superposition. On the contrary, in the same limit, one can show that $\mathcal{I}$ of ρ_m approaches zero, the same to that of $\rho_{th}(V \to \infty, 0)$, which is intuitively acceptable.

The apparent difference between the two measures shown in the above examples is due to the fact that $\mathcal{I}$ relates to $\dot{\mathcal{P}}$ while χ^2 to $\dot{\mathcal{P}}/\mathcal{P}$. We conclude that the purity in the denominator of χ^2 causes this measure to *overestimate* the degree of superposition for mixed states such as ρ_M and ρ_m.

We finally note that in order to guarantee positivity of $\mathcal{I}$, one may simply remove factor -1 in definition (1) of our Letter [1], and the modified definition is equivalent to $\mathcal{I} = (P - \dot{P})/2$ in the relevant decoherence model.

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[2] J. B. Gong, quant-ph arXiv:1106.0062.
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[5] H. Jeong and T. C. Ralph, Phys. Rev. Lett. **97**, 100401 (2006); Phys. Rev. A **76**, 042103 (2007).

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[7] We correct a typo in our Letter [1]: $RU - 4d^2(V+1)$ in Eq. (8) should be corrected to $RU + 4d^2(V+1)$.

[8] We discuss an extreme case here, but qualitatively the same conclusion is drawn from a finite $V \gg 1$.